Abstract: This paper continues a series of Vibration Institute papers on rotordynamics. Understanding Journal Bearings was published in 2001 covering the theory behind lubrication and fluid film bearings. Practical Rotor Dynamics was published in 2002 and discussed the general theory of rotor dynamics. In 2003 the paper Rotor Dynamics as a Tool for Solving Vibration Problems showed how rotor dynamics can be used to reduce vibration and make machinery more reliable. Very flexible rotors have critical speed problems with stability being a major concern. However, many rotors are not highly flexible and have different rotordynamic problems and concerns. Those are addressed in this paper. The dynamic effects of pedestal mass and stiffness are also explored. Overhung weights and moments have a significant effect on the vibration patterns of turbomachinery and behave differently from rotors where the major components are between bearings.

Key Words: Damping; Forced Response; Modeling; Overhung; Pedestal Effects; Properties; Rotors; Vibration

Introduction: The rigidity of a rotor is relative to the stiffness of the bearings supporting the rotor. Concentrating on only the first critical speed, consider two test rotors, figures 1 and 2. The first case, is a 4 inch diameter (D) shaft with a bearing span (L) of 40 inches. This gives a length-to-span ratio of 10-to-1 and a nominal stiffness of 273,000 LB/IN. The shaft stiffness (K) is calculated from Eq. 1 knowing the shaft dimensions and the material modulus (E) which is 29 X 10^6 for steel. In the second case the shaft diameter has been doubled to 8 inches for an L/D ratio of 5 and a shaft stiffness of 4,373,000 LB/IN. Thus, doubling the diameter results in a 16 times increase in shaft stiffness due to the inertia (I) increasing by the diameter to the fourth power.

\[ K_s = \frac{48 E I}{L^3} \]
\[ I = \frac{\pi D^4}{64} \]

Eq. 1

Increasing the shaft diameter looks like an ideal way to control the critical speed of a rotor. However, the rigid support first resonance, \( \omega_1 \) is related to both the shaft stiffness and the rotor mass, \( m \).

\[ \omega_1 = \sqrt{\frac{K_s}{m}} \]
\[ m = \frac{W}{g} \]

Eq. 2

Doubling the rotor diameter, increases its mass by a factor of 4. When the supports are rigid, only \( \frac{1}{2} \) the rotor mass contributes (see reference 1) so in eq. 2, \( m \) is reduced by half. Thus, in English units, the first rigid bearing critical speed, in CPM, is a constant (487,164) times the diameter, \( D \), divided by the span, \( L \), squared.
There are several unrealistic assumptions in this example. First, the rigid bearing critical speed has virtually no practical use. Secondly, real machines are more than just plain shafts with no overhangs on the ends and real rotors have blades, disks, impellers, etc. mounted. However, the basic goal here is to understand the difference between a “flexible” and a “rigid” rotor. There is a great deal to be learned by analyzing the two rotors shown above. Figure 3 is an undamped critical speed map similar to those shown in reference 1. The first critical speed is calculated for each rotor as a function of support stiffness.

The mode shape when the support stiffness is low looks like the rotor is bouncing up and down as in figure 4 and the relative amplitude at the bearing locations is high. When the bearings become very stiff, the amplitude at the support points approaches zero and there is significant shaft bending as in figure 5. It is important to note that the first and second critical speeds are not inherently bending modes although they are often mistakenly called bending modes. The third critical speed is the first inherently bending mode.

Figure 3 shows that, when the support stiffness is very low, the thicker rotor has a critical speed frequency that is \( \frac{1}{2} \) that of the slender rotor. **Under these conditions the stiffer rotor has a lower critical speed.** This means that the larger rotor’s mass effect is dominant since it weighs 4 times as much. For either rotor, in the low support stiffness range, the rotor first critical speed varies significantly as the support stiffness changes. As the supports become very stiff, the critical speed varies very little. The lines become asymptotic with the rigid bearing critical speed frequency. If a rotor-bearing system were designed to place it in the “flexible rotor”
region of the critical speed map then there would be no hope of altering the critical speed by stiffening the bearings. In this region, the only way to alter the critical speed is to alter the rotor stiffness or mass. Conversely, if the design is in the “rigid rotor” area of the critical speed map, the critical speed will be very sensitive to small changes in bearing stiffness caused by geometry differences like bearing clearance. The primary advantage of designing rotor systems to be in the “rigid rotor” area is that the available bearing damping will be many times more effective than when the bearings are so stiff that little relative motion is allowed.

Making the rotor stiffer will lower the critical speed with soft supports and raise the critical speed with stiff supports. At some support stiffness value, changing the rotor stiffness will not have any significant effect on the rotor critical speed. There is a range, labeled as “semi-flexible” or “semi-rigid” where the critical speed is affected equally by both the rotor stiffness and the shaft stiffness. This region is not exactly definable, but it seems that a lot of machines are produced that fall into this category.

The amplification factor is a measure of how well a dynamic system is damped. Damping is the mechanism that controls critical speed amplitude by transforming resonant energy to heat. Damping is difficult to add to existing systems but may be incorporated into original designs more easily. In turbomachinery the principal sources of damping are fluid film bearings and oil bushing seals. Other, less common sources are squeeze film dampers, certain types of gas seals, and friction devices (Coulomb damping). Critical damping ($C_C$) is that amount of damping in the system that will dissipate the most energy at resonance. It is defined in eq. 3.

$$C_C = 2m \omega_n = 2 \sqrt{K/m}$$

Eq. 3

For typical turbomachinery mounted in fluid film bearings, the percentage of critical damping ranges from about 5 to 50 percent. If we plot the amplification factor for our test rotors above as a function of the stiffness ratio $2K_b/K_s$ and apply 10 percent of critical damping we find that it makes no difference whether the rotor is slender or thick. The key is that it is much easier to achieve a lower stiffness ratio with a stiffer rotor.
Most machinery users consider an amplification greater than 10 to be unacceptable. While there are many machines running successfully with high amplification factors, they are almost always troublesome, high maintenance machines. With 10 percent critical damping, in order to keep the amplification factor to a tolerable level, the stiffness ratio $2K_m/K_s$ must be less than about 1.4. For the slender rotor this means individual bearing stiffnesses of 191,300 LB/IN or less. Unfortunately, in real machinery, a bearing this soft is very difficult to achieve. For the thicker rotor, the individual bearings would need to be less than 3 million LB/IN which is easily achievable. With a higher percentage of critical damping available, the bearings can be stiffer, but the same trend applies: it is much easier to control the critical speed amplitudes when the rotor is stiffer. Another thing that we can learn from figure 6 is that we always need more than 10 percent critical damping in order to achieve an amplification less than 2.5 at which point API has dictated that a resonance is no longer considered “critical”. In fact, about 50 percent of critical damping is required to reduce the amplification factor that much and that may not always be economically feasible.

If we vary the amount of damping, as a percentage of critical damping, then we can plot the critical speed unbalance response as in figure 7 for various percentages.

![Figure 7 - Unbalance Response of Slender Rotor and Various Damping Levels](image)

**Figure 7 - Unbalance Response of Slender Rotor and Various Damping Levels**

Figure 7 reveals several factors about the effects of damping. First, 100 percent of critical damping is not required to get acceptable vibration response. Second, since damping acts as a frequency dependent dynamic stiffness ($C_0 = \text{LB/IN}$), the critical speed frequency increases as the damping increases. Third, when the damping exceeds 100 percent of the critical value, the amplitude and, as seen in figure 8, the amplification factor increases. It is clearly more detrimental to have too little damping than too much, but anything between 50 percent and 125 percent yields similar results.

Thus, the old definition of a rigid rotor being one that “doesn’t pass through a critical speed” is very limited. The right way to classify a rotor is by how flexible it is compared to its supports. In general, greater rotor flexibility is not desirable although compromises are often needed depending on the requirements of a particular application. Lower support stiffness is beneficial to reduced amplification and increased damping effectiveness.
Figure 8 - Amplification Factor as a Function of Critical Damping

Figure 9 - Rigid Example L/D = 6
Figure 10 - Flexible Example L/D = 14

Figure 9 is typical of a real machine with a fairly rigid rotor. This turbine does pass through a critical speed with an amplification factor of 2.8 and is very well behaved with no instability problems. The compressor rotor in figure 10 has 9 stages and a long span. The first critical speed has an amplification factor that started out at 16 with the original bearings. After an optimized (softer) bearing was installed the amplification factor was reduced to 11 which is still fairly high. High vibration, rubs and instabilities were eliminated by providing additional bearing damping. These example rotors also show overhangs that are common to almost all machines. Whether these overhangs are couplings or governor parts or even the primary impeller, they can have a significant effect on the rotordynamics as discussed below.

Effects of Supports on Rotordynamics: Another factor that complicates rotordynamics is the interaction with the structure supporting the bearings. This machine element can be called the bearing pedestal, the machine casing or the foundation. This effect can be modeled as shown in figure 9 where a box representing the pedestal mass is shown in the middle of the support “spring”. The portion of the spring above the box represents the bearing stiffness and damping while the spring below the box is the stiffness and damping of the pedestal itself. Typically the pedestal to ground connection doesn’t have a lot of damping but it is important to include 2 to 10 percent of critical damping (calculated from the pedestal characteristics) in order to get good results that correlate to test data. In machines where the casing outweighs the rotor by more than 10-to-1 the substructure model can usually be ignored. However as the mass of the support substructure gets closer and closer to the rotor weight, it becomes paramount that it be included in the model.
Since the pedestals actually have a dynamic impedance, not just a static stiffness, the best way to include the effects of these supports is to conduct tests on the actual machine. An example is shown in figure 11 of an aero-derivative gas turbine engine used to drive oilfield injection pumps. The rotor weight and the case weight are nearly the same. Impact tests conducted on the case of this machine produced transfer function plots similar to figure 12. These values can be converted into dynamic stiffness and damping values. The rotordynamics computer program then applies the proper characteristics at each speed. This is often the only way to get good correlation with test data when the rotor weight and the casing weight are similar.

Figure 11 - Multi-Level Rotor Model with Case Flexibility and Pedestal Effects

Figure 12 - Transfer Function Test of Gas Turbine Engine Case
In order to illustrate the effect the supporting structure has on the dynamics of a rotor, the relative weight and the relative stiffness of the pedestal must be compared to the rotor weight and bearing stiffness. In order to simplify, 5 percent of critical foundation damping will be assumed for each case where critical damping is calculated from the pedestal mass and stiffness only. First, let’s go back to the slender rotor pictured in figure 1 and add pedestal effects as indicated by the boxes in the middle of the springs in figure 13.

If the support substructure is similar in mass and stiffness to the rotor, the first critical speed will be bifurcated or split-in-two. This is similar to what happens when a “vibration absorber” is applied. While that device would be more accurately called a system detuner, in essence a two degree-of-freedom system is created. The lower resonance has the mass of the rotor and the mass of the support moving together in-phase against the support stiffness. The higher frequency created has the mass of the rotor moving out-of-phase with the support mass and the effective spring rate is doubled. Figure 14 shows a Bodé plot calculated for this type of system.

The amplitude of the lower frequency resonance is less because the same forcing function (imbalance) is having to move twice as much mass. However, the amplification factor is higher since the support damping is dominant for this mode while the greater amount of bearing damping acts on the second resonance. This concept is detailed in reference 3 in non-dimensional terms and an actual case history. As the support mass increases, the lower resonant amplitude declines and its frequency approaches zero. In fact, for most turbomachinery, the support mass is at least 5 times the rotor mass. At this point, the support stiffness becomes
a concern. To illustrate this, the support stiffness of the steam turbine pictured in figure 9 was varied. The vibration was calculated for the shaft relative motion at the governor end displacement probe location. Figure 15 is a plot of the predicted vibration versus speed for this turbine when the pedestals are relatively soft (1,000,000 LB/IN) and when they are five times stiffer. Because of thermal growth allowances, pedestal stiffness of hot machinery like this turbine are more of an issue than machinery where the thermal expansion doesn’t have to be accommodated.

The pedestal flexibility effects seen in figure 15 fall into two main categories. As the structure supporting the bearings decreases in stiffness the critical speed frequency will also drop since the springs are in series. This could compromise the separation margin from operating speed required by API specifications. This also may explain why the critical speeds of existing machines may change over time. As parts wear or as repeated disassembly and reassembly reduce the tightness of register fits, the effective support stiffness may decrease significantly. The author has been asked many times to explain why a machine that has run well for decades has deteriorated and now is a problem. Often the only solution is remachining all the critical fits and restoring proper clearances - an expensive procedure.

The other consequence of a flexible pedestal support is a reduction in the effectiveness of the bearing damping. If one thinks of the oil film as a spring and a damper, picture the damper as an automobile type shock absorber. A shock absorber works by forcing oil from one chamber to another through an orifice as an internal piston is compressed. This absorbs energy that is converted to heat. Feeling the body of a car shock absorber after a bumpy ride will reveal that it is quite warm. If the shock absorber connection to ground is rigid then the damping will be effectively applied as the car (rotor) tries to move and compress the shock absorber (oil film). If the shock absorber is attached to ground via a weak spring then, as the rotor tries to push against it, the shock absorber just moves away and doesn’t compress and dissipate energy as designed.

While the influence of flexible pedestals can be extreme like the examples shown here, most of the time it is a secondary effect. However, all rotordynamics analyses should endeavor to include a substructure model that is as accurate as possible. There are also other ways that support flexibility can be introduced that will adversely affect the rotordynamics. References 6 and 7 have shown significant theory and practical application of including pedestal models in rotordynamics. In one important contribution, reference 8, Dr. John Nicholas discusses how different types of pivots for tilting pad bearings can impact rotor response.
Overhung rotors: There is a large group of machines that fall into this class. Most single stage pumps are overhung as well as many fans and compressors. The overhung format is popular because direct access to the suction side of a centrifugal impeller is available and makes the case design less complicated. In reality, most overhung machines are double-overhung with a coupling opposite the impeller end. The geometric variables involved include the bearing span, shaft diameter, and the span between the overhung masses and the nearest bearing. Many overhung machines like ANSI pumps mounted in rolling element bearings have little dynamic response since they usually operate well below the first critical speed. In those cases, the case mass and case stiffness are important to the ability of the machine to withstand the mechanical and operational forces imposed in normal operation. Case structural resonances can influence the vibration of these machines.

First, let’s look at a typical overhung machine as depicted in figure 16. This is a 3-inch diameter shaft with a 24 inch bearing span. The coupling on the left side is 40 pounds and 10 inches from the nearest bearing. On the other end the impeller weighs 200 pounds and is 12 inches from the nearest bearing.

In this configuration, nearly all the rotor weight is concentrated on the bearing nearest the impeller. In many cases the coupling end bearing will be loaded upwards. This is an important point and many engineering and maintenance procedures will be affected. It is not necessary to have a significant load on the coupling end bearing provided the bearing at that location is properly designed. The critical speed mode shapes of an overhung machine are different than the previous examples. The first and second modes are pivotal as seen in figures 17 and 18.

The first single overhung mode will always have the maximum amplitude at the impeller and the second will have maximum amplitude at the coupling. It is rare to encounter critical speeds above the second resonance in overhung machines as the frequencies get to be very high. It is extremely important that the overhung polar moments of inertia be accurately included as the **gyroscopic stiffening** can significantly raise critical speeds.
The first critical speed of an overhung machine depends on the impeller weight and the shaft diameters. In order to “tune” the critical speeds, a designer has limited options. The impeller design is usually fixed by the performance requirements and its weight can only be adjusted by using a less dense material like aluminum or titanium in lieu of a steel impeller. This option may not be feasible for all services. The shaft diameter between the impeller and the nearest bearing is generally the location of a seal. This may be a simple labyrinth or a much more complicated oil bushing or dry gas seal. The seal designers wish to keep the shaft diameter as small as possible in this area for greater sealing effectiveness and lower costs. This leaves the shaft section between the bearings as the candidate of choice for altering the first critical speed. It is not uncommon to find a larger diameter in this area and figure 19 is typical with a 6-inch diameter midspan.

![Figure 19 - Thick Midspan Example](image1)

The thicker section between bearings alters the first critical speed mode shape, figure 20, by severely reducing the shaft bending between bearings and increasing it between the bearing and the impeller. Plots 21 and 22 show how the thickness of the shaft between the bearings affects the critical speed and the amplification factor.

![Figure 20 - First Mode Shape - Thick Rotor](image2)

![Figure 21 - Effect of L/D Ratio on Critical Speed](image3)

![Figure 22 - Effect of L/D Ratio on Amplification Factor](image4)

The lower the span L/D ratio, the thicker the midspan diameter. When the L/D ratio in this example is 3, the diameter is 8 inches and when the ratio is 8, the shaft diameter is 3 inches (L=24). Increasing the midspan diameter increases the critical speed and lowers the amplification factor. However, this effect is limited as both curves show this effect becoming asymptotic. The increased shaft diameter will also reduce the amplitude at the impeller eye and possibly avoid rubs. However, increased midspan shaft diameter is not always the best design. In reference 11 the midspan diameter of an overhung compressor rotor was deliberately reduced to alter a critical speed. In that case the rigidity of the shaft was causing high amplitude at the impeller end bearing and rapid wear with bearing failures. The reduced diameter design placed the critical speed well below operating speed and reduced the transmitted force to the bearing by a factor of 5. Changing the midspan diameter also alters the bearing loading and could change the coupling end load direction. Often a bearing design change will be required if the shaft geometry is changed.
As noted above, the shaft section on an overhung machine between the bearing and the impeller is the location of a seal. This is also a section that may see significant pressure and temperature gradients. The seal length may be increased with higher pressures and an insulating barrier may be included as in the case of a cryogenic expander. Figures 23 and 24 show what effect changing the length of this section from 8 inches to 16 inches will have on the critical speed and amplification factor of our example machine.

![Figure 23 - Effect of Impeller to Bearing Distance on Critical Speed](image1)

![Figure 24-Effect of Impeller to Bearing Distance on Amplification Factor](image2)

As the length of this section increases, the critical speed drops rapidly and the amplification factor increases dramatically. The maximum displacement amplitude at the impeller eye will also increase by a factor of 5 as the length of this section doubles. These are serious design considerations for both new machines and those that may be modified.

**Double Overhung Machines:** When there are two impellers, as is the case in most integrally geared compressors, there is a slightly different double overhung effect. If we replace the coupling of the above example with an impeller (figure 25), we have a typical double overhung compressor rotor.

![Figure 25 - Double Overhung Rotor from Integrally Geared Compressor](image3)

The first stage impeller on the right is larger and heavier than the second stage impeller on the left by 25 pounds. The shaft is symmetric in dimensions. The first critical speed mode shape, figure 26, shows the two impellers vibrating out-of-phase with each other and the larger impeller has the highest relative amplitude. The second resonance mode shape, figure 27, shows the impellers vibrating in-phase with each other and now the lighter impeller has the highest relative amplitude. These resonances differ in frequency only slightly. The closer the impellers are to each other in weight, the closer the frequencies.
Typically, the bearings in this type of machine are identical and the gear load is significantly greater than the gravity load. Due to the difficulty in installing displacement probes near the impellers, they are usually placed next to the bearings or even on the center span section. This is not a good choice since the amplitude at the impeller eyes, where rubs occur, is an order of magnitude (or more) greater than that measured by the proximity probe. It is important to evaluate this when setting vibration limits on this type of machine. The synchronous unbalance response, figure 28, indicates the two different resonances are seen at the larger impeller but the lighter impeller response is dominated by the higher frequency resonance. This result is for a linear ratio of impeller weight to imbalance. As mentioned before, accurate inclusion of the overhung polar moments of inertia are paramount to accurately predicting the dynamic behavior of overhung machinery.

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REFERENCES


